

## 등온방사 공정에서 생기는 Draw Resonance에 미치는 변형속도에 따라 변하는 이완시간의 영향

현 재 천  
고려대학교 공과대학 화학공학과  
(1988년 11월 10일 접수)

### The Effect of Strain-rate Dependent Relaxation Time on the Draw Resonance of Isothermal Melt Spinning Processes

Jae Chun Hyun

*Department of Chemical Engineering, Korea University, Anam-Dong, Sungbuk-Ku, Seoul 136 - 701, Korea*

(Received November 10, 1988)

**요 약:** HDPE(고밀도 폴리에틸렌)과 LDPE(저밀도 폴리에틸렌)이 여러 폴리머 공정에서 근본적으로 서로 다른 거동을 보인다는것은 그동안 잘알려져 왔다. 예를 들어, 용융방사에서 draw resonance를 발생시키는 임계연신도( $r_c$ )의 값이 HDPE에서는 LDPE에 비해 현저하게 작고, 또한 물질의 이완시간( $\lambda$ )이 draw resonance에 미치는 영향도 서로 반대다. 본 논문에서는 HDPE와 LDPE의 이러한 두가지 다른 거동양태를, White의 변형속도에 따라 변하는 이완시간의 model을 사용하는 convected Maxwell 유체의 방사모형에, Hyun의 draw resonance 이론을 적용해서, 예측할수 있음을 보인다. Hyun의 운동학적 draw resonance 이론은 최근, 관계되는 편도함수를  $(x, t)$ 좌표계에서 수치계산으로 풀어본 바, 맞다는 결론이 나왔다. 이러한 수치모사연구의 결과는 따로 보고할 예정이다.

**Abstract:** It has long been known that the behavior of HDPE (high density polyethylene) and LDPE (low density polyethylene) in various polymer processing operations, is fundamentally different from each other. For example, in melt spinning, the critical draw-down ratio ( $r_c$ ) causing the onset of draw resonance, has much smaller values for HDPE than those for LDPE, and also the effect of the material relaxation time ( $\lambda$ ) on draw resonance is opposite in HDPE as compared to LDPE. In this paper, we show that these two different modes of the behavior of HDPE and LDPE can be predicted by using the Hyun's draw resonance theory on the spinning of convected Maxwell fluids with the White's strain-rate dependent relaxation time model. The Hyun's kinematical wave theory of draw resonance has recently been confirmed to be correct by the numerical calculation of the pertinent partial derivatives in the  $(x, t)$  coordinate system. The results of these numerical simulation studies will be reported elsewhere.

## INTRODUCTION

White and his research associates have studied both theoretically and experimentally the many behavioral differences among HDPE (high density polyethylene), LDPE (low density polyethylene), PP (polypropylene), PS (polystyrene) and others under various polymer processing conditions.<sup>1-5</sup> In order to interpret their interesting experimental results, they have used, as the constitutive equation, the model of convected Maxwell fluids (i. e., the White-Metzner model) along with a strain-rate dependent relaxation time.<sup>1,2</sup>

They found that the parameter "a" in their strain rate dependent relaxation time (which indicates the degree of the effect of the strain-rate on the material relaxation time), plays a very important role in distinguishing different polymers according to their different response to processing and rheological tests.

In particular, Minoshima and White recently reported<sup>3</sup> the results of many HDPE and LDPE samples in their behavior in melt spinning experiments in terms of "a",  $r_c$  (critical draw-down ratio at the onset of draw resonance), and  $\frac{\lambda V_0}{L}$  (dimensionless relaxation time or a Weissenberg number). In the present article, we explain how such results can be predicted analytically using the draw resonance theory by the present author.

The basic idea behind the draw resonance theory is, as explained in the original paper,<sup>6</sup> that the throughput waves travel from the spinneret to the take-up with a different velocity and a different traveling time (throughput residence time) from those of the fluid element (threadline residence time). Hence thus, the onset of draw resonance occurs when twice the throughput residence time exactly equals the threadline residence time as the draw-down ratio increases. (A positive wave and a negative one constitute one complete cycle of draw resonance in one threadline residence time.) We have recently confirmed that the above kinematical criterion of draw resonance is correct, by numerically calculating all the necessary partial

derivatives and the through-put wave residence time in the (x,t) coordinate system instead of (x, A) system.<sup>7</sup> We will explain more about this later in this article.

Since we are interested in the explanation of the different behavior of HDPE and LDPE in draw resonance as reported by Minoshima and White,<sup>5</sup> we use the same governing equations of isothermal melt spinning as before<sup>6</sup> (because we also used the same convected Maxwell fluids model then) but with the strain-rate dependent relaxation time (instead of a constant relaxation time used before) which includes the parameter "a".<sup>1,2</sup>

So our analytical results show that depending on the value of "a", not only  $r_c$  values change but also the question whether the relaxation time either stabilizes or destabilizes the melt spinning (i.e.,  $r_c$  either increases or decreases as the relaxation time increases) is answered. Our results corroborate White's data fairly well.

## DERIVATION OF THE RESULTS

First we review the results of Hyun<sup>6</sup> which used a constant relaxation time in the isothermal spinning of convected Maxwell fluids. Then we derive the analytical results for the same convected Maxwell fluids spinning using the White's strain-rate dependent relaxation time.

For the modeling of the system, we assume the same following conditions as before. 1) The secondary forces such as inertia, surface tension, and gravity are neglected. 2) The origin of the coordinate system starts at the die swell region. 3) The velocity distribution across the thread-line cross-section is uniform. 4) We consider stress, velocity, and so on, in the axial direction only.

Incorporating the above assumptions, we get a one-dimensional model which allows the analytical results.

Continuity equation:

$$\left( \frac{\partial A}{\partial t} \right)_x + \left| \frac{\partial (Av)}{\partial x} \right|_t = 0 \quad (1)$$

Equation of motion:

$$\frac{\partial}{\partial x} (A\sigma) = 0 \tag{2}$$

$$\text{and } \left(\frac{\partial \sigma}{\partial x}\right)_{s.s.} = \frac{d\sigma}{dx} = \left(\frac{F}{Q}\right) \left(\frac{\partial v}{\partial x}\right)_{s.s.}$$

Constitutive equation:

$$= \left(\frac{F}{Q}\right) \frac{dv}{dx} \tag{9}$$

$$\sigma + \lambda \left[ v \left(\frac{\partial \sigma}{\partial x}\right)_t - 2 \sigma \left(\frac{\partial v}{\partial x}\right)_t \right] = 2 \mu \left(\frac{\partial v}{\partial x}\right)_t \tag{3}$$

Substituting Eqs. (8) and (9) into Eq. (3), we get

Multiplying Eq. (1) by,  $\left[\frac{\partial(Av)}{\partial A}\right]_x$ , we obtain

$$\begin{aligned} \left(\frac{F}{Q}\right)v + \lambda \left[ v \left(\frac{F}{Q}\right) \frac{dv}{dx} - 2 \left(\frac{F}{Q}\right)v \frac{dv}{dx} \right] \\ = 2 \mu \frac{dv}{dx} \end{aligned} \tag{10}$$

a new continuity equation which yields the expression of the throughput (Av) wave traveling speed, U.

$$\left[\frac{\partial(Av)}{\partial t}\right]_x + U \left[\frac{\partial(Av)}{\partial x}\right]_t = 0 \tag{4}$$

$$\text{or } v = \frac{dv}{dx} (k + \lambda v) \tag{11}$$

$$U \equiv \left[\frac{\partial(Av)}{\partial A}\right]_x \tag{5}$$

$$\text{where } k = \frac{2\mu Q}{F} \tag{12}$$

Eq. (5) is further rewritten as follows.

$$\begin{aligned} \text{Thus } \left(\frac{\partial v}{\partial x}\right)_{s.s.} &= \left(\frac{\partial v}{\partial x}\right)_{Av} = \frac{dv}{dx} \\ &= \frac{V}{k + \lambda v} \end{aligned} \tag{13}$$

$$\begin{aligned} U \equiv \left[\frac{\partial(Av)}{\partial A}\right]_x &= - \frac{\left[\frac{\partial(Av)}{\partial x}\right]_A}{\left(\frac{\partial A}{\partial x}\right)_{Av}} \\ &= - \frac{A \cdot \left(\frac{\partial v}{\partial x}\right)_A}{\left(\frac{\partial A}{\partial x}\right)_{Av}} \end{aligned} \tag{6}$$

Hence, the denominator of Eq. (6) becomes

$$\begin{aligned} \left(\frac{\partial A}{\partial x}\right)_{Av} &= \left(\frac{\partial A}{\partial x}\right)_{s.s.} = \frac{dA}{dx} = \left(-\frac{(AV)}{V^2}\right)_x \\ \left(\frac{\partial v}{\partial x}\right)_{Av} &= -\frac{A}{V} \frac{dv}{dx} = -\frac{A}{(k + \lambda v)} \end{aligned} \tag{14}$$

The denominator in Eq. (6),  $\left(\frac{\partial A}{\partial x}\right)_{Av}$ , is obtained from the steady state solution of the system (i.e., when (Av) is held constant) whereas the numerator,  $\left(\frac{\partial v}{\partial x}\right)_A$ , is the key partial derivative we will talk about later.

Upon substitution of Eqs. (13) and (14) into Eq. (6), we obtain

First the steady state solution from Eqs. (1), (2) and (3) is as follows. (For more details, see Hyun & Ballman)<sup>8</sup>

$$\begin{aligned} U \equiv \left[\frac{\partial(Av)}{\partial A}\right]_x &= - \frac{A \left(\frac{\partial v}{\partial x}\right)_A}{\left(\frac{\partial A}{\partial x}\right)_{Av}} = - \frac{A \left(\frac{\partial v}{\partial x}\right)_A}{\frac{A}{(k + \lambda v)}} \\ &= (k + \lambda v) \left(\frac{\partial v}{\partial x}\right)_A \end{aligned} \tag{15}$$

$$Av = Q = \text{constant, and } A\sigma = F = \text{constant} \tag{7}$$

$$\text{and so } \sigma = \frac{F}{A} = \frac{F}{Q} v \tag{8}$$

The throughput wave traveling time from the spinneret (x=0) to the take-up (x=L), i. e., the

throughput residence time, is thus

$$t_L = \int_0^L \frac{dx}{U} = \int_0^L \frac{dx}{(k + \lambda v) \left( \frac{\partial v}{\partial x} \right)_A} \quad (16)$$

Now the fluid element traveling time, i. e., the threadline residence time, is calculated by

$$\begin{aligned} \tau_L &= \int_0^L \frac{dx}{V} = \int_{v_0}^{v_L} \frac{k + \lambda v}{v^2} dv \\ &= \frac{k}{v_0} \left( 1 - \frac{1}{r} \right) - \lambda (\ln r) \end{aligned} \quad (17)$$

since from Eq. (11),  $dx = \left( \frac{k + \lambda v}{v} \right) dv$  (18)

The expression for  $k$  in Eq. (17) can be obtained as follows.

Integrating Eq. (18) from  $x=0$  to  $x=x$ , we get

$$\begin{aligned} \int_0^x dx &= \int_{v_0}^v \frac{k + \lambda v}{v} dv, \text{ or} \\ x &= k \ln \frac{v}{v_0} + \lambda (v - v_0) \end{aligned} \quad (19)$$

That at the take-up,  $x=L$ , and  $v=v_r$ , leads to

$$\begin{aligned} L &= k \ln r + \lambda v_0 (r - 1), \text{ or} \\ k &= \frac{L - \lambda v_0 (r - 1)}{\ln r} \end{aligned} \quad (20)$$

Substitution of Eq. (20) into Eq. (17) produces

$$\tau_L = \frac{[L - \lambda v_0 (r - 1)] \left( 1 - \frac{1}{r} \right)}{v_0 \ln r} + \lambda (\ln r) \quad (21)$$

The criterion of the onset of draw resonance is then expressed as

$$2 t_L = \tau_L \quad (22)$$

where  $t_L$  and  $\tau_L$  are given by Eq. (16) and (21),

respectively. (The case of Newtonian fluids can be easily recovered by setting  $\lambda=0$  for all the above equations.)

In order to obtain  $r=r_c$  from Eq. (22), we need the expression for  $\left( \frac{\partial v}{\partial x} \right)_A$  for  $t_L$ . Previously, we used a linear approximation, i.e.,

$$\left( \frac{\partial v}{\partial x} \right)_A \approx \frac{v_L - v_0}{L} = \frac{v_0 (r - 1)}{L} \quad (23)$$

Then from Eq. (16) with the substitution of Eq. (23), we obtain

$$\begin{aligned} t_L &= \frac{L}{v_0 (r - 1)} \int_0^L \frac{dx}{k + \lambda v} = \left( \frac{L}{v_0 (r - 1)} \right) \times \\ &\int_{v_0}^{v_L} \frac{dv}{v} = \frac{L (\ln r)}{v_0 (r - 1)} \end{aligned} \quad (24)$$

Upon substitution of Eq. (21) and (24), the onset criterion on draw resonance, Eq. (22), becomes

$$\begin{aligned} \frac{2L (\ln r)}{v_0 (r - 1)} &= \frac{[L - \lambda v_0 (r - 1)] \left( 1 - \frac{1}{r} \right)}{v_0 (\ln r)} + \lambda (\ln r) \\ \text{or } \frac{2L (\ln r)^2}{(r - 1)} &= [L - \lambda v_0 (r - 1)] \left( 1 - \frac{1}{r} \right) \\ &\quad + \lambda v_0 (\ln r)^2 \end{aligned} \quad (25)$$

$$\text{Then } \tilde{\lambda} = \frac{\lambda v_0}{L} = \frac{2r (\ln r)^2 - (r - 1)^2}{(r - 1) [r (\ln r)^2 - (r - 1)^2]} \quad (26)$$

This is the result that we previously reported when we used an approximation shown in Eq.(23).

Before moving to the case of a strain-rate dependent relaxation time, we consider how to find the exact value of  $\left( \frac{\partial v}{\partial x} \right)_A$  unlike the approximation such as Eq. (23). The answer is the following relation which is derived by the coordinate transformation from  $(x,A)$  to  $(x,t)$ .

$$\left( \frac{\partial v}{\partial x} \right)_A = \left( \frac{\partial v}{\partial x} \right)_t + \left( \frac{\partial v}{\partial t} \right)_x \left( \frac{\partial t}{\partial x} \right)_A$$

$$= \left(\frac{\partial v}{\partial x}\right)_t - \left(\frac{\partial v}{\partial t}\right)_x \frac{\left(\frac{\partial A}{\partial x}\right)_t}{\left(\frac{\partial A}{\partial t}\right)_x} \quad (27)$$

Thus the numerical simulation of the system in the (x,t) coordinates, always yields the value of  $\left(\frac{\partial v}{\partial x}\right)_A$ . As Denn pointed out,<sup>9</sup> the above coordinate transformation doesn't hold when

$\left(\frac{\partial v}{\partial x}\right)_{x=0}$  (i. e., the vanishing Jacobian), which happens due to the oscillatory nature of A curves at any x with respect to t in draw resonance.

However, there are two reasons why we don't have to worry about  $\left(\frac{\partial A}{\partial x}\right)_{x=0}$  situations.

1) Numerically we can delete the points of

$\left(\frac{\partial A}{\partial t}\right)_{x=0}$  from the A curves, but still maintain the piecewise continuity of A(x,t) curves.

2) More importantly, since we are interested in  $t_L = \int \frac{dx}{U}$ , not in U itself, to obtain the value of  $r_c$  applying the criterion for the onset of draw resonance ( $2t_L = \tau_1$ ), the contribution of the points of  $\left(\frac{\partial A}{\partial t}\right)_{x=0}$  to the value of  $t_L$  is zero because those points make U infinite, which, in turn, makes  $1/U$  zero.

Therefore, the expression for  $\left(\frac{\partial v}{\partial x}\right)_A$  by Eq. (27) is easily usable in the numerical simulation which yields the value of  $r_c$  at the onset of draw resonance. We will report the results of the numerical simulation elsewhere, where among other things, the criterion of draw resonance (i.e., Eq. (22)) is proved, and the linear approximation (i. e., Eq. (23)) is shown to be a very good one.

Now we take the case of the strain-rate dependent relaxation time<sup>1,5</sup> in the spinning of the convected Maxwell fluids, and obtain the results similar to Eq. (26). All the governing equations are the same as those of the case of the constant relaxation time except the following two modifi-

cations. The first is the White's model of the strain-rate dependent relaxation time for extensional flow:

$$\lambda = \frac{\lambda_0}{1 + a\sqrt{3} \lambda_0 \left(\frac{\partial v}{\partial x}\right)_t} \quad (28)$$

where the parameter "a" indicates the degree of the strain-rate dependence of the relaxation time, and for shear flow  $\sqrt{3}$  is deleted from the denominator.

The second change is that  $\mu$  in the right hand side of Eq.(3) is replaced by Eq.(29)

$$\mu = G \lambda \quad (29)$$

where  $\lambda$  is given by Eq. (28), and  $G$ =modulus =constant.

Next, we follow the same procedure as before to get the results for the onset of draw resonance.

The constitutive equation, Eq. (3), with the substitution of Eqs. (28) and (29), becomes at steady state

$$\begin{aligned} \left(\frac{F}{Q}\right)V + \frac{\lambda_0}{1+a\sqrt{3} \lambda_0 \left(\frac{\partial v}{\partial x}\right)_t} \left| V \left(\frac{F}{Q}\right) \left(\frac{\partial v}{\partial x}\right)_t \right. \\ \left. - 2 \left(\frac{F}{Q}\right)V \left(\frac{\partial v}{\partial x}\right)_t \right| \\ = 2G \left(\frac{\partial v}{\partial x}\right)_t \frac{\lambda_0}{1+a\sqrt{3} \lambda_0 \left(\frac{\partial v}{\partial x}\right)_t}, \text{ or} \end{aligned}$$

$$V + \frac{dv}{dx} \left| a\sqrt{3} \lambda_0 v - \lambda_0 v - 2G\lambda_0 \left(\frac{Q}{F}\right) \right| = 0 \quad (30)$$

$$\begin{aligned} \text{Then } \frac{dv}{dx} &= \frac{V}{\lambda_0 | 2G(Q/F) + v(1-a\sqrt{3}) |} \\ &= \frac{V}{\lambda_0 | K + v(1-a\sqrt{3}) |} \quad (31) \end{aligned}$$

$$\text{where } K = \frac{2GQ}{F} \quad (32)$$

Also, 
$$\left(\frac{\partial A}{\partial x}\right)_{Av} = \frac{dA}{dx} = -\frac{A}{V} \frac{dv}{dx} = -\frac{A}{\lambda_0 [K + v(1 - a\sqrt{3})]} \quad (33)$$

and 
$$U \equiv -\frac{A\left(\frac{\partial v}{\partial x}\right)_A}{\left(\frac{\partial A}{\partial x}\right)_{Av}} = \lambda_0 [K + v(1 - a\sqrt{3})] \left(\frac{\partial v}{\partial x}\right)_A \quad (34)$$

Here we again as before use the approximation of  $\left(\frac{\partial v}{\partial x}\right)_A$ , i.e.,  $\left(\frac{\partial v}{\partial x}\right)_A \approx \frac{V_0(r-1)}{L}$ , for (34) because we want to derive analytical results and we also know this approximation is very close to the exact numerical solution at the onset of draw resonance.

Hence, 
$$U = \lambda_0 [K + v(1 - a\sqrt{3})] \frac{v_0(r-1)}{L}$$

and so, 
$$t_L = \int_0^L \frac{dx}{U} = \left(\frac{L}{v_0(r-1)}\right) \times \int_0^L \frac{dx}{\lambda_0 [K + v(1 - a\sqrt{3})]} = \frac{L}{v_0(r-1)} \int_{v_0}^{v_L} \frac{dv}{v} = \frac{L(\ln r)}{v_0(r-1)} \quad (35)$$

since from Eq.(31),

$$\frac{dx}{\lambda_0 [K + v(1 - a\sqrt{3})]} = \frac{dv}{V}$$

We can see that the expression of Eq. (35) is the same as Eq. (24), which implies that if we adopt a constant approximation for  $\left(\frac{\partial v}{\partial x}\right)_A$ , then the throughput wave traveling time becomes identical for all fluids; independent of the material functions such as  $\lambda$  and "a".

In order to find the expression for  $\tau_L$ , we again

use Eq. (31).

$$\begin{aligned} \tau_L &= \int_0^L \frac{dx}{V} = \int_{v_0}^{v_L} \frac{\lambda_0 [K + v(1 - a\sqrt{3})]}{v^2} dv \\ &= \frac{\lambda_0 K}{v_0} \left(1 - \frac{1}{r}\right) + \lambda_0 (1 - a\sqrt{3}) \ln r \end{aligned} \quad (36)$$

Finally, we substitute Eqs. (36) and (35) into Eq. (22) to get the relation between  $r$  and  $\lambda$ .

$$\begin{aligned} \frac{2L \ln r}{v_0(r-1)} &= \frac{\lambda_0 K}{v_0} \left(1 - \frac{1}{r}\right) \\ &+ \lambda_0 (1 - a\sqrt{3}) \ln r \end{aligned} \quad (37)$$

where  $K$  is obtained as follows.

From Eq. (31), integration leads to

$$\begin{aligned} \int_0^x dx &= \int_{v_0}^v \frac{\lambda_0 [K + v(1 - a\sqrt{3})]}{V} dv, \text{ or} \\ x &= \lambda_0 K \ln \frac{v}{v_0} + \lambda_0 (1 - a\sqrt{3}) (v - v_0) \end{aligned} \quad (38)$$

Then at the take-up,  $x=L$  and  $v_L=v_L$ , we get Eq.(39)

$$\begin{aligned} L &= \lambda_0 [K \ln r + (1 - a\sqrt{3}) v_0 (r - 1)] \text{ , or} \\ K &= \frac{1}{\lambda_0 \ln r} [L - \lambda_0 (1 - a\sqrt{3}) v_0 (r - 1)] \end{aligned} \quad (39)$$

Upon substitution of Eq. (39) into Eq. (37), we obtain the final result.

$$\begin{aligned} \frac{2L \ln r}{v_0(r-1)} &= \frac{1}{v_0 \ln r} [L - \lambda_0 (1 - a\sqrt{3}) v_0 (r - 1)] \\ &+ \lambda_0 (1 - a\sqrt{3}) \ln r \text{ , or} \\ \frac{2(\ln r)^2}{r-1} &= \left(1 - \frac{1}{r}\right) + \frac{\lambda_0 v_0}{L} (1 - a\sqrt{3}) \\ &\left[ (\ln r)^2 - (r-1) \left(1 - \frac{1}{r}\right) \right] \end{aligned} \quad (40)$$

Then  $\tilde{\lambda}_0 = \frac{\lambda_0 v_0}{L}$

$$= \frac{2r(\ln r)^2 - (r-1)^2}{(1-a\sqrt{3})(r-1)[r(\ln r)^2 - (r-1)^2]} \quad (41)$$

Comparing this with the case of the constant relaxation time shown in Eq. (26), we can see that the only difference is the factor  $\frac{1}{(1-a\sqrt{3})}$  the case of strain-rate dependent relaxation time.

### DISCUSSION

However, the plot of  $r$  vs  $\tilde{\lambda}$  according to Eq. (41) is quite different from that by Eq. (26). Fig. 1 is the case of the constant relaxation time by Eq. (26), and Fig. 2 shows the results by Eq. (41). Obviously, the case of  $a=0$  in Fig. 2 represents the case in Fig. 1.

Unlike the curve in Fig. 1 representing the case of the constant relaxation time, Fig. 2 shows two different modes of behavior of draw resonance of the convected Maxwell fluids spinning depending on the value of "a" in that for the fluids having  $a < 1/\sqrt{3} < 0.577$ , Fig. 1 type curves are obtained, whereas for the fluids having  $a > 1/\sqrt{3}$  differ-

rent type of curves where  $r_c$  decreases with the increasing  $\tilde{\lambda} = \frac{\lambda V}{L}$ . In other words, the effect of the relaxation time on the onset of draw resonance is opposite to each other depending upon the value of "a"

According to the reports by White, et. al.,<sup>1-5</sup> LDPE melts generally belong to the group I fluids having lower "a" values ( $a < 1/\sqrt{3}$ ), whereas HDPE melts belong to the group II fluids having higher "a" values ( $a > 1/\sqrt{3}$ ). Thus we can expect that Fig. 1 type behavior is exhibited by LDPE, while the behavior like the lower curves in Fig. 2 can be found with HDPE.

Specifically, the recent results by Minoshima and White<sup>7</sup> revealed larger values of  $r_c$  for LDPE (material having a lower "a" value) and smaller values of  $r_c$  for HDPE (material having a higher "a" value) at the onset of draw resonance. The results also showed that the dimensionless relaxation time,  $\tilde{\lambda} = \frac{\lambda V}{L}$ , stabilizes the spinning of LDPE, and destabilizes that of HDPE. (Namely, with the increasing,  $\tilde{\lambda}$ ,  $r_c$  increases for LDPE and decreases for HDPE). So the effect of the material relaxation time,  $\lambda$ , and of the spinning condition

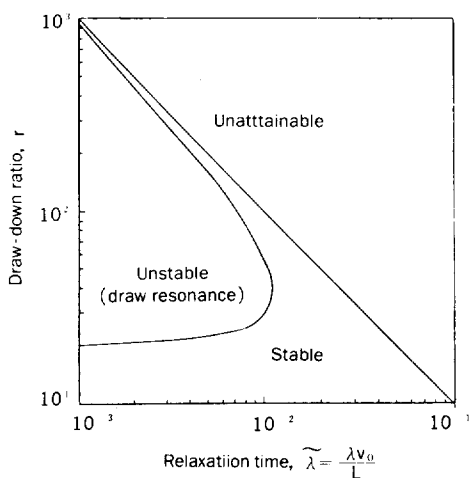


Fig. 1. Stability region for the spinning of Maxwell fluids.

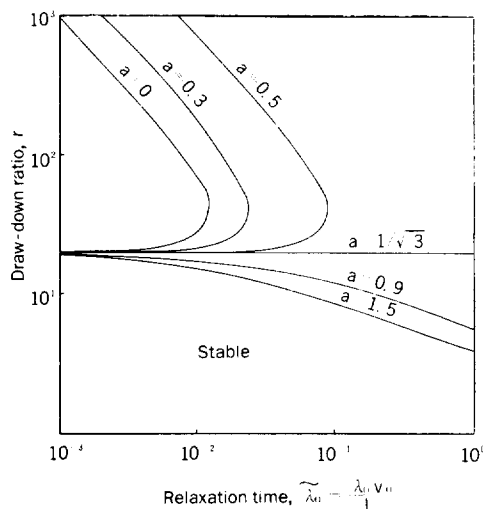


Fig. 2. Stability region for the spinning of Maxwell fluids having varying relaxation times.

(i. e.,  $v$  and  $L$  here) on the onset of draw resonance, was shown to be opposite to each other depending upon whether the fluid belongs to group I ( $a < 1/\sqrt{3}$ ) or group II ( $a > 1/\sqrt{3}$ ).

Hence, the predicted behavior of draw resonance shown in Fig. 2 corroborates their data fairly well. Their data also show that not all LDPE / HDPE samples belong to group I / group II, respectively and some samples definitely show the opposite behavior. So in this article, we use the words of LDPE / HDPE in the sense that they strictly represent group I / group II fluids, respectively and not to mean to cover all kinds of LDPE / HDPE samples exhibiting the opposite behavior. Some similar results have been reported using a more complex model than ours through a numerical calculation.<sup>10-13</sup>

## CONCLUSIONS

The simple model of the spinning of convected Maxwell fluids with the White's strain-rate dependent relaxation time, readily produces two different modes of behavior at the onset of draw resonance depending on the value of the material parameter "a" when we use the Hyun's kinematical draw resonance theory. This parameter "a" imbedded in White's relaxation time model, representing the strain-rate dependence of the material relaxation time, dichotomizes the fluids into group I type and group II type fluids. The fact that in general, LDPE belongs to the group I and HDPE to the group II, yields a good agreement between our theoretically predicted results and Minoshima and White's experimental data.

The Hyun's draw resonance theory has recently been confirmed to be correct by numerically calculating the through-put wave velocity (i.e., throughput residence time) and the fluid element velocity (i.e., threadline residence time) in the (x,t) coordinate system, and then comparing them in the draw resonance criterion equation.

We also used the linear approximation of the

key partial derivative,  $(\frac{\partial v}{\partial x})_A$  which turns out to be a very accurate one at the onset of draw resonance. Then we were able to obtain analytical results involving  $r_c$ ,  $\lambda$ , "a" and so forth, unlike other numerical studies.

Therefore, the utility of our approximation  $(\frac{\partial v}{\partial x})_A$  that we can easily check the experimental data with known physics<sup>15,16</sup> of the system (i.e., the concept of wave residence time) without going through complex numerical calculations, because of the analytical nature of our results,

Other polymer processing operations<sup>3-5, 14</sup> are amenable to our approximate yet accurate method which produces analytical results. Of course, the pursuit of more accurate constitutive models (e. g., multiple relaxation times, power-law type viscosity and so on) to characterize many real polymers, should be continued for the sake of the advancement of polymer rheology, but the power of analytical results is always indefatigable.

## NOTATION

- A = threadline cross-sectional area
- $A_0$  = threadline cross-sectional area at  $x=0$
- $A_L$  = threadline cross-sectional area at  $x=L$
- a = the material parameter of fluids as defined by Eq.(28)
- F = threadline tension
- G = material modulus
- $K = 2GQ / F$
- $k = 2\mu Q / F$
- L = distance from the spinneret to the take-up
- Q = throughput =  $A v$
- r = draw-down ratio
- $r_c$  = critical draw-down ratio at the onset of draw resonance
- t = time
- $t_L$  = throughput traveling (residence) time
- U = throughput wave velocity
- V = threadline (fluid) velocity



$V_0$  = threadline (fluid) velocity at  $x=0$   
 $V_L$  = threadline (fluid) velocity at  $x=L$   
 $x$  = distance from the spinneret  
 $\lambda$  = relaxation time of Maxwell fluids  
 $\tilde{\lambda}$  = dimensionless relaxation time =  $\lambda V_0 / L$   
 $\lambda_0$  = constant relaxation time of  $\lambda$ , when  $a=0$   
 $\tilde{\lambda}_0$  = dimensionless relaxation time =  $\lambda_0 V_0 / L$   
 $\mu$  = viscosity  
 $\sigma$  = stress  
 $\tau_L$  = threadline traveling (residence) time

### REFERENCES

1. Ide, Y. & J. L. White, *J. Non-N. Fluid Mech.*, **2**, 281 (1977).
2. Ide, Y. & J. L. White, *J. Appl. Polym. Sci.*, **22**, 1061 (1978).
3. Kanai, T. & J. L. White, *Polym. Eng. Sci.*, **24**, 1185 (1984).
4. White, J. L. & A. Kondo, *J. Non-N. Fluid Mech.*, **3**, 41 (1978).
5. Minoshima, W. & J.L.White, *J. Non-N. Fluid Mech.*, **19**, 251 & 275 (1986).
6. Hyun, J. C., *AIChE J.*, **24**, 418 (1978).
7. Hyun, J. C. & S. J. Lee, In preparation.
8. Hyun, J. C. & R. L. Ballman, *J. Rheology*, **22**, 349 (1978).
9. Denn, M.M., *AIChE J.*, **26**, 292 (1980).
10. Fisher, R. J. & M.M. Denn, *AIChE J.*, **22**, 236 (1976).
11. Chang, J. & M. M. Denn, *Rheology*, vol. 3, ed. by G. Astarita, et. al., 9 (1980).
12. Ishihara, H. & S. Kase, *J. App. Polym. Sci.*, **19**, 577 (1975).
13. Ishihara, H. & S. Kase, *J. Appl. Polym. Sci.*, **20**, 169 (1976).
14. Petrie, C. J. S. & M.M.Denn, *AIChE J.*, **22**, 209 (1976).
15. Mewis, J. & C. J.S. Petrie, *Encyclopedia Fluid Mechanics*, ed. by N.P. Cheremisinoff, **6**, 111 (1987).
16. Schultz, et. al., *J. Fluid. Mech.*, **149**, 445 (1984)